

Physics 1A Discussion (Week 1)

Problem 1 (problem 1.39 on page 29 of the textbook)

Given two vectors $\vec{A} = -2.00\hat{i} + 3.00\hat{j} + 4.00\hat{k}$ and $\vec{B} = 3.00\hat{i} + 1.00\hat{j} - 3.00\hat{k}$, where $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors (Here, the unit vectors $\hat{i}, \hat{j}, \hat{k}$ are defined so that each is normalized by 1 in length and points toward the positive direction of the x axis, y axis, and z axis, respectively.)

(a) Find the magnitude of each vector.

Since the inner product of the vector itself gives the squared magnitude(length) of the vector, we get

$$|\vec{A}|^2 = \vec{A} \cdot \vec{A} = (-2.00)^2 \hat{i} \cdot \hat{i} + (3.00)^2 \hat{j} \cdot \hat{j} + (4.00)^2 \hat{k} \cdot \hat{k} = 29 \Rightarrow |\vec{A}| = \sqrt{29} \quad (1)$$

$$|\vec{B}|^2 = \vec{B} \cdot \vec{B} = (3.00)^2 \hat{i} \cdot \hat{i} + (1.00)^2 \hat{j} \cdot \hat{j} + (-3.00)^2 \hat{k} \cdot \hat{k} = 19 \Rightarrow |\vec{B}| = \sqrt{19} \quad (2)$$

(b) Use unit vectors $\hat{i}, \hat{j}, \hat{k}$ to write an expression for the vector difference $\vec{A} - \vec{B}$.

$$\begin{aligned} \vec{A} - \vec{B} &= (-2.00 - 3.00)\hat{i} + (3.00 - 1.00)\hat{j} + [4.00 - (-3.00)]\hat{k} \\ &= -5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k} \end{aligned} \quad (3)$$

(c) Find the magnitude of the vector difference $\vec{A} - \vec{B}$.

From (b), we got $\vec{A} - \vec{B} = -5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k}$, then

$$\begin{aligned} |\vec{A} - \vec{B}|^2 &= (-5.00)^2 \hat{i} \cdot \hat{i} + (2.00)^2 \hat{j} \cdot \hat{j} + (7.00)^2 \hat{k} \cdot \hat{k} = 78 \\ \Rightarrow |\vec{A} - \vec{B}| &= \sqrt{78} \end{aligned} \quad (4)$$

(d) Is this the same as the magnitude of $\vec{B} - \vec{A}$? Explain.

Since $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, you get

$$\vec{B} - \vec{A} = (-1) \left(-5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k} \right) = 5.00\hat{i} - 2.00\hat{j} - 7.00\hat{k}, \quad (5)$$

then the length squared is given by

$$\begin{aligned} |\vec{B} - \vec{A}|^2 &= (5.00)^2 \hat{i} \cdot \hat{i} + (-2.00)^2 \hat{j} \cdot \hat{j} + (-7.00)^2 \hat{k} \cdot \hat{k} = 78 \\ \Rightarrow |\vec{B} - \vec{A}| &= \sqrt{78} \\ \Rightarrow |\vec{B} - \vec{A}| &= |\vec{A} - \vec{B}|. \end{aligned} \quad (6)$$

Visually, the vector $\vec{B} - \vec{A}$ is a vector with the same length as $\vec{A} - \vec{B}$ and points toward the opposite direction of the $\vec{A} - \vec{B}$.

$$\begin{array}{c} \vec{A} - \vec{B} \longrightarrow \\ \longleftarrow \vec{B} - \vec{A} \end{array}$$

Problem 2(An Earthlike Planet: problem 1.51 on page 30 of the textbook)

In January 2006, astronomers reported the discovery of a planet comparable in size to the Earth, orbiting another star and having a mass about 5.5 times the Earth's mass M_{Earth} . It is believed to consist of a mixture of rock and ice, similar to Neptune. If this planet has the same density as Neptune (1.76 g/cm^3), what is its radius expressed Necessary astronomical data are the following:

$$1\text{km} = 10^3\text{m} = 10^5\text{cm}, \quad 1\text{g} = 10^{-3}\text{kg} = 0.001\text{kg}, \quad M_{\text{Earth}} = 5.97 \times 10^{24}\text{kg}, \quad R_{\text{Earth}} = 6.37 \times 10^3 \text{ km}$$

(a) in km (kilometers)?

Let V_{planet} be the volume of the planet. If we assume the planet is the perfect sphere, then the volume can be written by

$$V_{\text{planet}} = \frac{4}{3}\pi R_{\text{planet}}^3, \quad (7)$$

where R_{planet} is the size(radius) of the planet.

Besides, the mass of the planet $M_{\text{planet}} = 5.5M_{\text{Earth}}$ is written in terms of the product of the density and the volume

$$\begin{aligned} M_{\text{planet}} &= (\text{density of the planet}) \times V_{\text{planet}} \\ \Rightarrow V_{\text{planet}} &= \frac{M_{\text{planet}}}{(\text{density of the planet})} \\ &= \frac{5.5 \times M_{\text{Earth}}}{(\text{density of Neptune})} \\ &= \frac{5.5 \times (5.97 \times 10^{24} \text{ kg})}{1.76 \text{ g/cm}^3} \\ &= \frac{5.5 \times (5.97 \times 10^{27} \text{ g})}{1.76 \text{ g/cm}^3} \\ &\sim 1.87 \times 10^{28} \text{ cm}^3. \end{aligned} \quad (8)$$

Since there is a unit conversion relation

$$1 \text{ km}^3 = (1 \text{ km})^3 = (10^3 \text{ m})^3 = (10^3 \times (10^2 \text{ cm}))^3 = (10^5 \text{ cm})^3 = 10^{15} \text{ cm}^3, \quad (9)$$

we get $V_{\text{planet}} \sim 1.87 \times 10^{28} \text{ cm}^3 = 1.87 \times 10^{13} \text{ km}^3$. Therefore we have

$$\begin{aligned} R_{\text{planet}}^3 &= \frac{3}{4\pi} V_{\text{planet}} \sim 4.46 \times 10^{12} \text{ km}^3 \\ \Rightarrow R_{\text{planet}} &\sim 1.65 \times 10^4 \text{ km} \end{aligned} \quad (10)$$

(b) as a multiple of earth's radius R_{Earth} ?

$$\begin{aligned} \frac{R_{\text{planet}}}{R_{\text{Earth}}} &\sim \frac{1.65 \times 10^4 \text{ km}}{6.37 \times 10^3 \text{ km}} \sim 2.58 \\ \Rightarrow R_{\text{planet}} &\sim 2.58 R_{\text{Earth}} \end{aligned} \quad (11)$$

Problem 3 (problem 1.68 on page 31 of the textbook)

You live in a town where the streets are straight but are in a variety of directions. On Saturday you go from your apartment to the grocery store by driving 0.60 km due north and then 1.40 km in the direction 60.0° west of north. On Sunday, you again travel from your apartment to the same store, but this time by driving 0.80 km in the direction 50.0° north of west and then in a straight line to the store.

(a) How far is the store from your apartment?

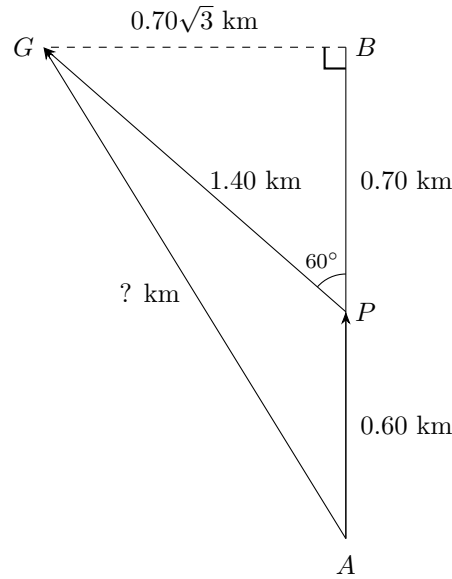


Figure 1: Illustration of the first path on Saturday

Fig 1 shows the path on the first day. We know the angle of $\angle GPB$ to be 60° , then from the trigonometric relation, we get

$$PB = PG \cos(60^\circ) = 1.40 \text{ km} \times \left(\frac{1}{2}\right) = 0.70 \text{ km} \quad (12)$$

$$BG = PG \sin(60^\circ) = 1.40 \text{ km} \times \left(\frac{\sqrt{3}}{2}\right) = 0.70\sqrt{3} \text{ km} \quad (13)$$

When we perform the Pythagorean theorem on $\triangle ABG$, we yield

$$\begin{aligned} AG^2 &= (0.60 \text{ km} + 0.70 \text{ km})^2 + (0.70\sqrt{3} \text{ km})^2 \\ &= 3.16 \text{ km}^2 \\ \Rightarrow AG &\sim 1.78 \text{ km} \end{aligned} \quad (14)$$

(b) On which day do you travel the greater distance, and how much farther do you travel? Or, do you travel the same distance on each route to the store?

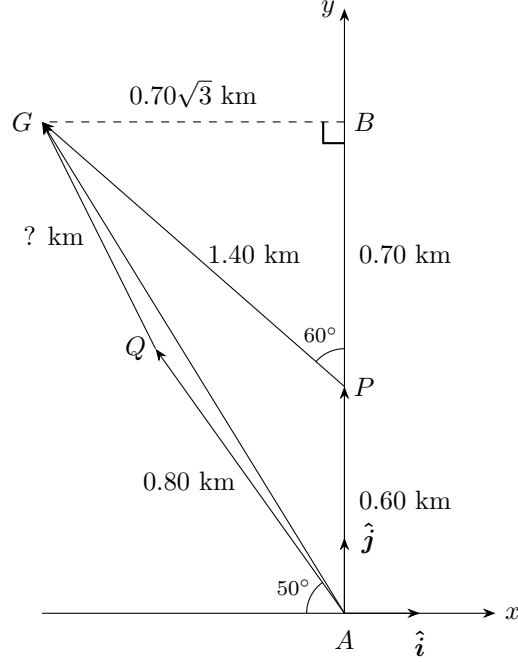


Figure 2: Illustration of the 2 paths drawn on the xy plane

Fig 2 shows the 2 paths in Cartesian coordinates. By using the unit vectors $\hat{i}, \hat{j}$ shown in this figure, you can express $\vec{AQ}$ and $\vec{AG}$ as

$$\vec{AQ} = (-0.80 \cos(50^\circ) \text{ km}) \hat{i} + (0.80 \sin(50^\circ) \text{ km}) \hat{j} \quad (15)$$

$$\vec{AG} = (-0.70\sqrt{3} \text{ km}) \hat{i} + (1.30 \text{ km}) \hat{j}. \quad (16)$$

Vector has a property of addition: $\vec{AG} = \vec{AQ} + \vec{QG}$ (*i.e.* only the displacement matters), then we get

$$\begin{aligned} \vec{QG} &= \vec{AG} - \vec{AQ} \\ &= [-0.70\sqrt{3} \text{ km} + 0.80 \cos(50^\circ) \text{ km}] \hat{i} + [1.30 \text{ km} - 0.80 \sin(50^\circ) \text{ km}] \hat{j} \\ &\sim (-0.70 \text{ km}) \hat{i} + (0.69 \text{ km}) \hat{j}. \end{aligned} \quad (17)$$

Therefore the length of QG is $|\vec{QG}| \sim \sqrt{(-0.70 \text{ km})^2 + (0.69 \text{ km})^2} \sim 0.98 \text{ km}$.

Finally, we compare the 2 paths. The total length of the first path is $L_1 = AP + PG = 0.60 \text{ km} + 1.40 \text{ km} = 2.00 \text{ km}$, while the total distance of the second path is $L_2 = AQ + QG \sim 0.80 \text{ km} + 0.98 \text{ km} = 1.78 \text{ km}$. From this calculation, we see that the first path L_1 is longer than the second path L_2 by

$$\Delta L = L_1 - L_2 \sim 0.22 \text{ km} \quad (18)$$